# Discriminating MSSM families in (free-field) Gepner orientifolds 

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Abstract: A complete analysis of orientifold compactifications involving Gepner models that are free fields $(\mathrm{k}=1,2)$ is performed. A set of tadpole solutions is found that are variants of a single chiral spectrum. The vacua found have the property that different families have different $U(1)$ charges so that one family cannot obtain masses in perturbation theory. Its masses must come from instantons, allowing for a hierarchy of masses. The phenomenological aspects of such vacua are analyzed.

Keywords: Intersecting branes models, Conformal Field Models in String Theory, Superstring Vacua.

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## 1. Introduction and conclusions

The search for vacua of string theory that resemble the SM has a 24 year history, and is ongoing. In the last decade, orientifold vacua attracted a lot of attention in this respect as it became understood that they allow a bottom-up approach [1-3] in assembling the SM ingredients. There are many distinct ways of embedding the Standard Model group into that of quiver gauge theories, which appear in the context of orientifolds and these are reviewed in [4]-77. A general framework for classifying such embeddings in orientifolds, in particular that of the hypercharge, was developed in [8] based on some mild assumptions. This framework was applied to orientifolds that can be constructed from Gepner models (studied earlier in (9-15), using the algorithmic techniques of RCFT developed in 16.

A total of 19345 chirally distinct top-down spectra were found, that comprise so far the most extensive such list known in string theory, [B]. For 1900 of these spectra at least one tadpole solution was also found. Combined with earlier results for vacua realizing the Madrid incarnation [17] of the Standard Model, (15] they contain the largest collection of vacua (tadpole solutions) chirally realizing the (supersymmetric) SM.

Unfortunately, further progress in this direction is hampered by the fact that the tools to calculate the superpotential and other important low energy quantities are not yet so well developed.

In this paper we will focus on a small subset of such vacua that share a simplifying property: their CFTs and BCFTs can be constructed out of free fields. It is known that there are two Gepner models that are equivalent to free fields. The $k=1$ model is equivalent to a free boson with $c=1,18,19$ while the $k=2$ model is equivalent to a free boson and an Ising fermion with central charge $c=3 / 2$ [20]. There are several ways of tensoring these two models in order to construct an orientifold compactification. We find that only vacua made out of six copies of the $k=2$ model have the potential to produce spectra that resemble those of the SM. It is such vacua that we will focus on this paper. For the other tensor combinations of free field $\mathrm{N}=2$ minimal models, namely $(1,1,1,2,2,2,2),(1,1,1,1,1,1,2,2)$ and ( $1,1,1,1,1,1,1,1,1$ ) not even a SM configuration without tadpole cancellation (i.e. the analog of a local model) was found in [8].

Our goal here is two-fold. First to make a detailed and extensive search for orientifold vacua that are chirally similar to the supersymmetric SM. ${ }^{1}$ Second, to provide a qualitative phenomenological study of the tadpole solutions found, in order to assess their potential to provide phenomenologically acceptable and interesting realizations of the SSM. If both of the above goals are achieved successfully, the road is open to a detailed calculation of the effective potential and interactions.

Our results are summarized as follows:

- There are 96 tadpole solutions found in the $(k=2)^{6}$ compactification that all realize the chiral spectrum No. 14062 in the classification of [8]. They give rise to 8 distinct massless spectra. There are two possible hidden sector gauge groups: $\mathrm{Sp}(2)$ and $\mathrm{O}(2)$. The eight spectra differ apart from the hidden sector gauge group also in the non-chiral spectrum of massless particles.
- If we relax the assumption made in [8] that no chiral observable-hidden matter is present, then we find three more chirally distinct spectra, Nos. 101, 559, 800 in the list of [8]. These include Pati-Salam models but we will not study them further in this paper.
- Only the tadpole solutions with a hidden $\operatorname{Sp}(2)$ group have a phenomenologically sufficient number of right-handed neutrinos.
- There are three $\mathrm{U}(1)$ gauge symmetries, two of which are free of fourdimensional anomalies and one is "anomalous". One of the two non-anomalous ones

[^0]is hypercharge. The other has a massive gauge boson and is therefore expected to be violated by string-instanton effects.

- In order for this solution to be phenomenologically viable, other points in its moduli space must be chosen, so that the massless non-chiral exotics obtain sufficiently high masses in order to satisfy experimental constraints.
- One of the three families has different charges under the two "anomalous" $\mathrm{U}(1)$ symmetries compared to the other two. This has as a consequence that selection rules for low energy couplings are in effect. In particular, this family remains massless in perturbation theory.
- There is a single pair of Higgs multiplets
- A $\mu$-term is allowed and must therefore be tuned to small values.
- To protect low-energy lepton number conservation discrete symmetries must operate. Baryon number is violated only by $\mathrm{SU}(2)_{\text {weak }}$ instantons.
- The Fayet-Iliopoulos terms appearing in the low-energy potential are shown to be zero at the tadpole solution point. They must be kept zero as we move in moduli space. As a byproduct we generalize to arbitrary CFTs/BCFTs previous proofs on the vanishing of loop corrections to the FI terms provided tadpoles cancel.
- String instanton corrections are necessary (and are classified) in order for the third family to acquire masses.
- The expected pattern of the neutrino mass matrix is of the see-saw type allowing for light neutrino masses.
- Although the branes are not in a "unified" configuration, $\sin ^{2} \theta_{W}=\frac{6}{13}$ at the string scale and differs by less than $20 \%$ from the unified value of $\frac{3}{8}$. Therefore, a change in the masses of the charged non-chiral massive particles can accommodate a conventional "unification" of gauge couplings.
- The strong dynamics of the hidden non-abelian gauge group can trigger supersymmetry breaking. However, to obtain an acceptable scale, appropriate threshold corrections must be advocated just below the string scale.

Although the results indicate that this class of vacua are potentially compatible with phenomenology, this requires also several special conditions to be met. A lot of detailed analysis is necessary in order to achieve this and we hope to report on this in a subsequent publication.

## 2. The tadpole solutions of the Gepner $(k=2)^{6}$ orientifold SM

In this paper we consider the tensor product of six $N=2$ minimal superconformal field theories with $k=2$. The central charge of each factor is $\frac{3}{2}$, so that the internal CFT
has $c=9$, equivalent to six free bosons and fermions. Each $k=2$ factor has 24 primary fields. Each factor is equivalent to the tensor product of a free boson with 8 primaries and an Ising model. This means that the resulting CFT can be realized in terms of free fields, in contrast to most other $N=2$ minimal model tensor products (a.k.a. Gepner models). However, in the construction of modular invariant partition functions (MIPFs), orientifolds and tadpole solutions no use is made of the specific free field theory properties of these models. After tensoring the six factors with the space-time NSR fermions, imposing world-sheet supersymmetry by extending the chiral algebra with the product of all worldsheet supersymmetry generators, and extending the chiral algebra to obtain space-time supersymmetry, we end up with a CFT with 2944 primary fields, 512 of which are simple currents. Under fusion, these simple currents close to form a discrete group $\mathbf{Z}_{4} \times \mathbf{Z}_{4} \times \mathbf{Z}_{4} \times \mathbf{Z}_{2}$.

We now build all the MIPFs that can be constructed using these simple currents, using the algorithm of 21, 22]. In normal circumstances all these MIPFs would be distinct, but in this case there are two special circumstances: a permutation symmetry among the six identical factors, and the fact that each factor contains an Ising model. A special feature of the Ising model is that its simple current MIPF is identical to the diagonal invariant. This happens because the only simple current orbit with charge $\frac{1}{2}$ happens to be a fixed point of the simple current (this orbit is formed by the spin field of the Ising model). This degeneracy extends to products of Ising models, and as a result some generically distinct MIPFs are actually identical.

The permutation symmetry occurs frequently in other Gepner models, and we deal with it by considering only one member of a permutation orbit. The Ising degeneracy occurs only in a few cases and can be dealt with by comparing the resulting MIPFs. The only problem is that there is some interference between the two degeneracies. It may happen that an Ising degeneracy does not occur between the selected representatives of the permutation orbits, but between other members. This will then result in some overcounting.

Although this degeneracy can be removed in principle, ${ }^{2}$ we have not implemented this because the overcounting is only a minor problem. After removing permutations and identical MIPFs we end up with 1032 MIPFs, and we expect the actual number of distinct ones to be slightly smaller than this. For each MIPF we construct all simple current orientifolds, according to the prescription of [16]. The total number of distinct orientifolds (taking into account known orientifold equivalences as described in 16] and the permutation symmetry) ranges from 4 to 64 , depending on the MIPF. This includes some zero-tension orientifolds that are of no further interest, since the dilaton tadpole forbids all Chan-Paton multiplicities.

For each MIPF we then compute all boundary states, using the formula given in 16. To each of these cases we then search for standard model configurations. Here we apply the same search algorithm used already in 8 for the other Gepner models. The only difference is that we remove the upper limit on the number of boundary states, which was set at

[^1]1750 in [8] for purely practical reasons. In the case of the $2^{6}$ model, only a handful of MIPFs exceed that limit, and therefore we decided to do a complete scan. This did not yield anything new, though. Indeed, the standard model configurations we describe below were all already found during the search performed in [8].

The last step in the procedure is to try and solve the tadpole conditions for the hidden sector, in order to cancel all tadpoles introduced by the orientifold and the Standard Model configuration. Here too we went slightly beyond [8] by allowing chiral matter between the observable and the hidden sector. Normally this produces such a huge number of solutions that it is preferable to require observable/hidden matter to be non-chiral. While chiral observable/hidden matter is not necessarily a phenomenological disaster (and can even be desirable in certain circumstances), it does require additional mechanisms to make it acquire a mass. In this particular case, however, we already knew that the number of tadpole solutions was extremely small, so it seemed worthwhile to try and relax the criteria.

The search of [8 produced a total of about 19000 chirally distinct standard model configurations, and tadpole solutions were found for 1900 of them. In the new search for the $2^{6}$ model we found tadpole solutions for 4 models. On the list of 19000 (ordered according to the first time each spectrum occurred, ${ }^{3}$ and available on request) these were nrs. 101, 559, 800 and 14062. Only in the latter case did we find solutions with non-chiral observable/hidden matter. This means that this last case was within the scope of [8]. Nevertheless, the tadpole solutions were not found at that time for a very simple reason: no attempt was made to solve the tadpole conditions for a certain model if a solution was already known. In this particular case, there turns out to exist a solution for spectrum nr. 14062 for Gepner model $(2,2,2,6,6)$, which was found first. It was presented in $\|$ in section 6.5, as a "curiosity". This model is rather similar to the ones presented here, but the $(2,2,2,6,6)$ model is not a free CFT. It is in fact so similar (including non-chiral matter, which is not taken into account when comparing spectra) that we expect that these models are actually related, presumably by an orbifold procedure that maps three copies of $k=2$ to two copies of $k=6$, but we have not investigated this.

In all orientifolds of all MIPFs of the tensor product $2^{6}$ the spectrum 14062 occurred 168 times, and in 96 cases there was a solution to the hidden sector tadpole equations. These solutions occurred for the following MIPF numbers: 41, 414, 415, 416, 417, 418, $644,646,651,652,662,1018,1021$. These numbers are labels assigned by the generating program "kac" to the 1032 MIPFs, and are listed here in order to identify the MIPFs and reproduce them, if necessary.

For comparison we give here the total number of boundary state configurations with at least one tadpole solution for the other models: 43008 for nr. 800,168 for nr. 559 and 6144 for nr . 10 . Note that this is not the total number of tadpole solutions: any given boundary state configuration may admit many, often a huge number, of tadpole solutions. We only explored the full set of solutions for spectra of type 14062. As already mentioned above, all tadpole solution for spectrum types 800,559 and 101 contain chiral observable-hidden matter. For these three configurations there are no tadpole solutions without such chiral

[^2]| MIPF id | Order | $h_{11}$ | $h_{12}$ | Singlets | Sol. Types | Total | Glob. An? |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 41 | 32 | 11 | 17 | 223 | 1112 | 4 | No |
| 414 | 32 | 17 | 11 | 223 | 3345 | 4 | No |
| 415 | 32 | 11 | 17 | 223 | 1112 | 4 | No |
| 416 | 32 | 17 | 11 | 223 | 3345 | 4 | Yes |
| 417 | 32 | 9 | 15 | 219 | 6678 | 16 | Yes |
| 418 | 32 | 11 | 17 | 223 | 1112 | 4 | Yes |
| 644 | 128 | 11 | 17 | 223 | 1112 | 4 | No |
| 646 | 128 | 9 | 15 | 219 | 6678 | 12 | Yes |
| 651 | 128 | 17 | 11 | 223 | 3345 | 4 | No |
| 652 | 128 | 17 | 11 | 223 | 3345 | 4 | Yes |
| 662 | 128 | 9 | 15 | 219 | 6678 | 12 | Yes |
| 1018 | 32 | 9 | 15 | 219 | 6678 | 8 | Yes |
| 1021 | 32 | 9 | 15 | 219 | 6678 | 16 | No |

Table 1: The MIPFs with tadpole solutions
exotics. On the other hand, for spectrum 14062 all tadpole solutions are free of chiral exotics. In fact, in some cases there is no observable-hidden matter at all.

Spectrum 14062 has a Chan-Paton group $\mathrm{U}(3) \times \mathrm{Sp}(2) \times \mathrm{U}(1) \times \mathrm{U}(1)$, with the hypercharge realized as in the familiar "Madrid" configuration 17], but with an interchange of the rôles of brane cand d for some of the quarks and leptons. In contrast to the Madrid models, which with very rare exceptions have an exact $B-L$ gauge symmetry, all superfluous $\mathrm{U}(1)$ 's in these models are broken, so that the surviving gauge symmetry (apart from the hidden sector) is exactly $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$. We will discuss these spectra in much more detail in the next section. Usually we will denote the Chan-Paton factor $\operatorname{Sp}(2)$ as $\mathrm{SU}(2)$ when its orientifold origins are unimportant.

Table (1) lists the main characterizations of the MIPFs for which tadpole solutions for spectrum 14062 exist. We specify the order of the simple current subgroup that produces them, the Hodge numbers of the compactification and the number of singlets in the spectrum for the corresponding heterotic string theory. The number of boundary states is 320 in all cases, and the gauge group in the heterotic theory is $E_{6} \times E_{8} \times \mathrm{U}(1)^{5}$ in all cases. Of course the orientifolds we construct are based on a type-IIB theory, and heterotic data are only given here as a way to characterize the MIPF.

The simple current group is $\mathbf{Z}_{4} \times \mathbf{Z}_{4} \times \mathbf{Z}_{2}$ if the order is 32 and $\mathbf{Z}_{4} \times \mathbf{Z}_{4} \times \mathbf{Z}_{2} \times \mathbf{Z}_{2} \times \mathbf{Z}_{2}$ if the order is 128 . Note that the list of Hodge numbers is not mirror symmetric. The complete list of Hodge numbers of the $2^{6}$ tensor product is mirror symmetric, even if one includes the number of singlets and gauge bosons. However, mirror symmetry does not extend to the boundary states, indeed not even to the total number of boundary states. Nevertheless, there do exist MIPFs with Hodge data $(15,9,219)$ and even precisely 320 boundary states, but they did not produce any solutions.

Columns 6 and 7 specify some information concerning the tadpole solutions we found.

| Spectrum | H | $Y_{A}$ | $Y_{S}$ | $P_{A}$ | $P_{S}$ | $R$ | $T$ | $X$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\mathrm{Sp}(2)$ | 0 | 4 | 0 | 2 | 2 | 1 | 4 |
| 2 | $O(2)$ | 0 | 4 | 0 | 2 | 3 | 0 | 0 |
| 3 | $O(2)$ | 4 | 0 | 0 | 2 | 1 | 2 | 4 |
| 4 | $\mathrm{Sp}(2)$ | 4 | 0 | 0 | 2 | 0 | 3 | 4 |
| 5 | $O(2)$ | 4 | 0 | 0 | 2 | 1 | 2 | 0 |
| 6 | $O(2)$ | 2 | 2 | 2 | 0 | 1 | 2 | 4 |
| 7 | $\mathrm{Sp}(2)$ | 2 | 2 | 2 | 0 | 0 | 3 | 4 |
| 8 | $O(2)$ | 2 | 2 | 2 | 0 | 1 | 2 | 0 |
| $(2,2,2,6,6)$ | $\mathrm{U}(2)$ | 4 | 0 | 0 | 2 | 0 | 0 | 0 |

Table 2: The distinct spectra and their non-chiral exotics. The first eight occur in the (2, 2, 2, 2, 2, 2) tensor product and are the subject of this paper. The last one has been found in $\| \beta$ for the $(2,2,2,6,6)$ tensor product.

In column 7 we indicate for how many standard model configurations at least one solution exists. It turns out that in each of those cases (i.e. 96 in total) there are in fact four solutions to the tadpole conditions, one with a hidden sector gauge group $\mathrm{Sp}(2)$, and three with an $O(2)$ hidden sector group. Of the total number of $4 \times 96=384$ solutions only 8 are different. In column 6 we indicate which of those eight solutions occur for each MIPF. This turns out to depend only on the MIPF, and not on the standard model configuration. Note that the kind of solution that occurs correlates perfectly with the Hodge data.

The eight distinct spectra are tabulated in table (2). All eight spectra have identical chiral states, which we specify in the next section. Here we just focus on the differences, which consist of the choice of hidden sector gauge groups, and some non-chiral exotics. Column two lists the hidden gauge group $H$. The other columns specify the multiplicities of the seven kinds of non-chiral exotics that may occur. We have named them $Y_{A} \ldots X$, and in table (3) we indicate their Chan-Paton representations. For comparison we have also listed the $(2,2,2,6,6)$ model presented in [8] in table (2). It has an $\mathrm{U}(2)$ hidden sector group with the rare feature of being completely hidden, by not having any massless matter at all (of course there do exist massive excited states in all open string sectors). Note also that all these spectra, including the last, have the same total number of non-chiral rank-2 exotics for each of the a,b,c and d branes, which may be distributed in different ways over symmetric and anti-symmetric representations.

An important additional constraint is the absence of global anomalies. In RCFT models, this leads to a large number of necessary conditions obtained by adding probe branes to a given model, as discussed in [23]. Since the probe branes at our disposal are limited by "rationality" of the RCFT, it is not guaranteed that this exhausts all possible origins of global anomalies, but we do take into account all the ones we can. In Gepner orientifolds these constraints eliminate some models, but their effect is limited to rather few tensor combinations, and is not extremely restrictive even in those cases 24]. Also in the present class there turn out to be tadpole solutions with global anomalies, but they

| $\mathrm{U}(3)_{a}$ | $\mathrm{SU}(2)_{b}$ | $\mathrm{U}(1)_{c}$ | $\mathrm{U}(1)_{d}$ | $H$ | Y | Symbol |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 0 | 0 | 0 | 0 | $\pm \frac{1}{3}$ | $Y_{A}$ |
| S | 0 | 0 | 0 | 0 | $\pm \frac{1}{3}$ | $Y_{S}$ |
| 0 | 0 | 0 | A | 0 | 0 | $P_{A}$ |
| 0 | 0 | 0 | S | 0 | $\pm 1$ | $P_{S}$ |
| 0 | 0 | 0 | 0 | A | 0 | $R$ |
| 0 | 0 | 0 | 0 | S | 0 | $T$ |
| 0 | 0 | V | 0 | V | $\pm \frac{1}{2}$ | $X$ |

Table 3: The non-chiral exotics that may occur in the eight distinct models.
were already eliminated from the set discussed above. In column 8 of table 1 we indicate in which cases there were additional tadpole solutions with global anomalies. Note that these anomalous solutions do not correlate with the Hodge data.

## 3. The low-energy characteristics of tadpole solution No. 1

All of the tadpole solutions we presented in the previous section are missing two righthanded singlets in the SM stack. Overall SM singlets, even if they do not come from the SM stack can in principle play the role of right-handed neutrinos. A look at table 2 shows that global singlets with zero mass are the multiplets labeled $R$ for the hidden $\mathrm{Sp}(2)$ group $^{4}$ or the multiplets $T$ for the hidden $\mathrm{SO}(2)$.

It is preferable for phenomenological reasons (supersymmetry breaking in particular) to a have a strongly-coupled gauge group in the hidden sector. The presence of a sufficient number of right-handed neutrinos ${ }^{5}$ and the requirement of a non-abelian hidden sector therefore selects spectrum No. 1, which has a hidden $\operatorname{Sp}(2)$ group. The complete spectrum of this solution is shown in table 4.

The solutions we find have unbroken $\mathrm{N}=1$ supersymmetry in four dimensions, therefore each entry of table $\pi^{7}$ corresponds to an $\mathrm{N}=1$ chiral multiplet. The $\mathrm{N}=1$ vector multiplets for all gauge groups are assumed. As usual $V$ stands for the vector representation, $V^{*}$ for the conjugate vector representation, $S$ for the two-index symmetric representation while $A$ stands for the two-index antisymmetric representation. In particular for a $\mathrm{U}(1)$ gauge group, $V$ indicates charge $+1, V^{*} \rightarrow-1, S \rightarrow+2$, while $A$ indicates a missing massless particle (although the associated stringy tower is intact as the projection alternates at alternate string levels). Dimension gives the total number of multiplets independent of

[^3]| Dim | Chirality | $\mathrm{U}(3)_{a}$ | $\mathrm{SU}(2)_{b}$ | $\mathrm{U}(1)_{c}$ | $\mathrm{U}(1)_{d}$ | $\mathrm{SU}(2)_{h}$ | Y | Symbol |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 3 | V | V | 0 | 0 | 0 | $+\frac{1}{6}$ | Q |
| 2 | -2 | V | 0 | $\mathrm{~V}^{*}$ | 0 | 0 | $+\frac{2}{3}$ | $U$ |
| 2 | -2 | V | 0 | V | 0 | 0 | $-\frac{1}{3}$ | $D$ |
| 3 | -1 | V | 0 | 0 | $\mathrm{~V}^{*}$ | 0 | $\pm \frac{2}{3}$ | $\mathcal{U}$ |
| 1 | -1 | V | 0 | 0 | V | 0 | $-\frac{1}{3}$ | $\mathcal{D}$ |
| 2 | 2 | 0 | V | 0 | V | 0 | $-\frac{1}{2}$ | L |
| 3 | 1 | 0 | V | V | 0 | 0 | $\pm \frac{1}{2}$ | K |
| 3 | -3 | 0 | 0 | V | V | 0 | -1 | $E_{R}$ |
| 1 | 1 | 0 | 0 | V | $\mathrm{~V}^{*}$ | 0 | 0 | $N_{R}$ |
| 4 | 0 | S | 0 | 0 | 0 | 0 | $\pm \frac{1}{3}$ | $Y_{S}$ |
| 2 | 0 | 0 | 0 | 0 | S | 0 | $\pm 1$ | $P_{S}$ |
| 4 | 0 | 0 | 0 | V | 0 | V | $\pm \frac{1}{2}$ | $X$ |
| 2 | 0 | 0 | 0 | 0 | 0 | A | 0 | $R$ |
| 1 | 0 | 0 | 0 | 0 | 0 | S | 0 | $T$ |

Table 4: The massless spectrum of tadpole solution No. 1 of spectrum 14062.
chirality, while Chirality gives the net chiral number of multiplets. Chirality is + by convention for left-handed fermions and its minus for left-handed fermions. Dimension=3, Chirality $=3$ therefore means that there are 3 left-handed multiplets. while dimension $=3$, chirality $=-1$ means there are 2 right-handed and one left-handed multiplets.

The hypercharge tabulated in table 4 is given by

$$
\begin{equation*}
Y=\frac{1}{6} Q_{3}-\frac{1}{2} Q_{c}-\frac{1}{2} Q_{d} \tag{3.1}
\end{equation*}
$$

whose gauge boson is massless in this solution. ${ }^{6}$ This is the Madrid hypercharge embedding or $x=\frac{1}{2}$ in the global classification of [8].

The following massless states are charged (non-chiral) exotics beyond the MSSM:

- A pair of the up-like anti-quarks $\overline{\mathcal{U}}$.
- The 2 right-handed and 2 left handed 6 representations of $\mathrm{SU}(3)$, labelled $Y_{s}$ in table 8 Although they have fractional hypercharge, all colour singlets that one can make using them have integer electric charge.
- The 2 right-handed and 2 left handed multiplets labelled $X$ in table 0 . They are doublets of the hidden $\mathrm{SU}(2)_{h}$, and have half-integer Y and electric charge.
- The 1 right-handed and 1 left handed multiplet labelled $P_{s}$ in table 7 . They have charge $\pm 2$ under $\mathrm{U}(1)_{d}$ and have integer Y and electric charge.

[^4]|  | $\mathrm{U}(1)_{3}$ | $\mathrm{U}(1)_{c}$ | $\mathrm{U}(1)_{d}$ |
| :---: | :---: | :---: | :---: |
| $\mathrm{SU}(3)_{a}$ | 0 | 0 | 0 |
| $\mathrm{SU}(2)_{b}$ | 9 | 1 | 2 |
| $\mathrm{SU}(2)_{h}$ | 0 | 0 | 0 |
| gravity | 0 | 0 | 0 |

Table 5: The mixed four-dimensional anomalies of $U(1)$ 's

- The one real multiplet labelled $T$ in table $T^{4}$ transforming as the adjoint of the hidden $\mathrm{SU}(2)_{h}$ group.

Finally we should stress that the two chiral multiplets labeled $R$ in table $\theta^{2}$ are absolute singlets (as the antisymmetric of $\mathrm{SU}(2)_{h}$ is a singlet) and are expected to play the role of the missing 2 right-handed neutrinos.

Because of the above fields the particular point in the moduli space where the tadpoles were solved is not suitable for describing the low-energy world. It is natural to assume that by moving a distance of order of the string scale in moduli space such non-chiral states will acquire masses which may be anywhere from 100 TeV to the string scale so they are directly unobservable. Of course such particles may have indirect effects in the low energy physics. Below we will consider all possible non-renormalizable superpotential terms and therefore we are sure to include all indirect effects due to these massive states.

Therefore in the sequel we will assume that the multiplets $Y_{s}, X, P_{s}, T$ and one non-chiral pair of the $\overline{\mathcal{U}}$ quarks are massive and have been integrated out.

### 3.1 Anomalies

It is by now well known that generic $\mathrm{U}(1)$ gauge symmetries in orientifold vacua are anomalous. Their anomalies are canceled by the GS mechanism that in four dimensions involves closed string axion scalars [25. In the process, the associated gauge bosons acquire a mass that is generically moduli dependent [26, 27] and the gauge symmetry is broken. Unless the associated global symmetry is also spontaneously broken by D-terms, it survives in perturbation theory and is only broken by gauge instantons.

It is important to stress that a $\mathrm{U}(1)$ gauge symmetry can be broken and its associated gauge boson acquires a mass even when the $\mathrm{U}(1)$ in question has no four-dimensional anomalies. This phenomenon was observed in [17, 26] and was explained in [27].

In the vacuum at hand we can calculate the four-dimensional mixed anomalies of the three $\mathrm{U}(1)$ factors. The results are in table 5 .

The anomaly matrix is defined $K_{I J}=\operatorname{Tr}\left[Q_{J}\left(T^{a} T^{a}\right)_{I}\right]$, where $J=3, c, d$, and $I=$ 1 corresponds to the colour $\mathrm{SU}(3), I=2$ corresponds to the weak $\mathrm{SU}(2)$, and $I=3$ corresponds to the mixed gravitational anomaly $\operatorname{Tr} Q_{J}$.

- Note that the only non-trivial non-abelian anomaly is that with $\mathrm{SU}(2)_{b}$. This implies that there are two independent $\mathrm{U}(1)$ combinations that are free from four-dimensional anomalies. We find however that only one of them, the hypercharge in (3.1) is massless. Therefore, all other $\mathrm{U}(1)$ 's except Y are massive.
- $\mathrm{U}(1)_{a}$ is baryon number and it is violated only by $\mathrm{SU}(2)_{b}$ instantons. This violation is tiny and therefore baryon number is a very good global symmetry of this vacuum 28.
- None of the two $\mathrm{U}(1)$ 's that are anomaly free in four dimensions $\left(a Q_{a}+c Q_{c}+d Q_{d}\right.$ with $9 a+c+2 d=0$ ) is violated by gauge instantons. Y remains massless and we expect no violation due to instantons. However we expect that the other anomaly-free $\mathrm{U}(1)$ symmetry it is broken by stringy instantons.
- Although there are anomalous $\mathrm{U}(1)$ 's and mixed anomalies, the gravitational mixed anomaly is zero.

This vacuum has two extra anomalous $\mathrm{U}(1) \mathrm{s}$ beyond the SM symmetries. Extra (anomalous) $\mathrm{U}(1)$ symmetries are a generic prediction of orientifold vacua, their number ranging from a minimum of one to several, [1], 29]. The masses of such gauge bosons can be low when the string scale is low. They can also be accidentally low even if the string scale is large in the case of highly asymmetric compactifications, [26]. The phenomenological consequences of anomalous $\mathrm{U}(1)$ gauge bosons in such cases have been explored in [29-31. A review on Z's from string theory can be found in 32.

## 4. The low energy MSSM fields

After integrating out the non-chiral exotics we are left with fields that are in one to one correspondence with the MSSM. ${ }^{7}$ We have 3 quarks $Q^{I}$, two up and down anti-quarks $U^{i}$, $D^{i}$, of the first type, one anti-quark of the second type: $\mathcal{U}$, one down anti-quark of the second type $\mathcal{D}$, two lepton doublets, $L^{i}$, two left handed lepton doublets $K^{i}$ that together with the right-handed doublet $\bar{H}$ will provide the third lepton double and the pair of MSSM Higgs, three right-handed electrons $E^{I}$ and three (neutrino) singlet $N$ and $R^{i}$. They are all summarized in table 6 along with their various $\mathrm{U}(1)$ charges.

There are two immediate observations. A $\mu$-term $K^{i} \bar{H}$ is not forbidden by the gauge symmetry in the superpotential but we are at a special point where this term is zero. There are two possibilities: (a) either this term is forbidden by one of the discrete symmetries of the vacuum or (b) this term is moduli dependent, and we happen to be at one of its zeros. In any case we will assume that we are in a region of moduli space that this term is small compared to the string scale and close to what is required for electro-weak physics.

The second observation is that because one of the lepton doublets (orthogonal to the one that mixes with $\bar{H}$ ) has exactly the same quantum numbers, (including the anomalous $\mathrm{U}(1)$ charges) as the Higgs, we expect the lepton number to break at the renormalizable level. In this theory, baryon number as we will discuss later is expected to be a very good global symmetry as only $\mathrm{SU}(2)$ gauge instantons break it. ${ }^{8}$ Because of this the constraints on lepton number violation are weak, but exclude however renormalizable couplings. To

[^5]| Number | $\mathrm{U}(1)_{3}$ | $\mathrm{SU}(2)_{b}$ | $\mathrm{U}(1)_{c}$ | $\mathrm{U}(1)_{d}$ | Y | Chiral field |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 1 | $\mathbf{2}$ | 0 | 0 | $+\frac{1}{6}$ | $\mathrm{Q}^{I}$ |
| 2 | -1 | $\mathbf{1}$ | -1 | 0 | $-\frac{2}{3}$ | $U^{i}$ |
| 2 | -1 | $\mathbf{1}$ | 1 | 0 | $+\frac{1}{3}$ | $D^{i}$ |
| 1 | -1 | $\mathbf{1}$ | 0 | -1 | $-\frac{2}{3}$ | $\mathcal{U}$ |
| 1 | -1 | $\mathbf{1}$ | 0 | 1 | $+\frac{1}{3}$ | $\mathcal{D}$ |
| 2 | 0 | $\mathbf{2}$ | 0 | 1 | $-\frac{1}{2}$ | $L^{i}$ |
| 2 | 0 | $\mathbf{2}$ | 1 | 0 | $-\frac{1}{2}$ | $K^{i}$ |
| 1 | 0 | $\overline{\mathbf{2}}$ | -1 | 0 | $+\frac{1}{2}$ | $\bar{H}$ |
| 3 | 0 | $\mathbf{1}$ | -1 | -1 | +1 | $E^{I}$ |
| 1 | 0 | $\mathbf{1}$ | 1 | -1 | 0 | $N$ |
| 2 | 0 | $\mathbf{1}$ | 0 | 0 | 0 | $R^{i}$ |

Table 6: The low energy MSSM states as left-handed chiral multiplets
proceed we will now write all quadratic and cubic terms in the superpotential that are allowed by the gauge symmetries both anomalous and non-anomalous.

The most general gauge-invariant quadratic superpotential is

$$
\begin{equation*}
W_{2}=K \bar{H}+R R \tag{4.1}
\end{equation*}
$$

while the cubic one is

$$
\begin{equation*}
W_{3}=Q U K+Q D \bar{H}+Q \mathcal{U} L+L N \bar{H}+L E K+K \bar{H} R+R R R \tag{4.2}
\end{equation*}
$$

where we have dropped both the indices and coefficients as we are interested in the qualitative features.

The following observations are relevant

- A linear term in $R$ is allowed in the superpotential as $R$ is a global singlet. This term is zero in the Gepner point, but may appear in other regions of moduli space and along with gaugino condensation may trigger supersymmetry breaking.
- It is reasonable to assume that the role of Higgses is taken over by $\bar{H}$ and a linear combination of $K^{i} .9$
- The anti-quarks $\mathcal{U}, \mathcal{D}$ have no Yukawa coupling in $W_{3}$.
- Because the Higgs $H$ and one of the leptons have the same global quantum numbers, several couplings violate lepton number.

Looking further we may write down the most general quartic superpotential consistent with the gauge symmetry,

$$
\begin{align*}
W_{4}= & (Q U)(L N)+(Q D)(L E)+(Q U)(Q D)+(Q \mathcal{U})(Q \mathcal{D})+(L L)(E N)+K \bar{H} K \bar{H} \\
& +(Q \mathcal{D}) \bar{H} N+(Q \mathcal{D}) E K+K \bar{H} R R+W_{3} R \tag{4.3}
\end{align*}
$$

[^6]We observe that if the right-handed neutrino $N$ obtains a vev, the $\mathcal{D}$ quark (not to be confused with the two quarks $D$ ) acquires a Yukawa coupling, which will be very small for any acceptable value of the vev of $N$.

## 5. Lepton number violation and discrete symmetries

To avoid lepton number violation at the observable level a discrete symmetry must be invoked. This discrete symmetry must distinguish between the two chiral doublets $K^{1}, K^{2}$ that will provide one Higgs and one lepton doublet. There may be several such discrete symmetries but the one that will do the job is the following $Z_{2}$ symmetry

$$
\begin{equation*}
K^{1} \leftrightarrow K^{2} \quad, \quad L^{i} \rightarrow-L^{i} \quad, \quad E^{I} \rightarrow-E^{I} \quad, \quad N \rightarrow-N \quad, \quad R^{i} \rightarrow-R^{i} \tag{5.1}
\end{equation*}
$$

If we now label $K^{1}+K^{2} \rightarrow H$ which will now be the Higgs and $K^{1}-K^{2} \rightarrow \mathcal{L}$ which will now be the third lepton doublet, we may rewrite the superpotentials that are invariant under such a symmetry

$$
\begin{align*}
W_{2}= & H \bar{H}+R R  \tag{5.2}\\
W_{3}= & Q U H+Q D \bar{H}+L N \bar{H}+L E H+\mathcal{L} R \bar{H}  \tag{5.3}\\
W_{4}= & (Q U) L N+(Q D) L E+(Q U)(Q D)+(Q \mathcal{U})(Q \mathcal{D})+L L E N+ \\
& +\mathcal{L} \overline{\mathcal{H}} \bar{H}+H H \bar{H} \bar{H}+(Q \mathcal{D}) E \mathcal{L}+H \bar{H} R R+(Q U) \mathcal{L} R+(Q \mathcal{U}) L R+ \\
& +L E \mathcal{L} R+H \bar{H} R R+R R R R \tag{5.4}
\end{align*}
$$

We observe that

- Lepton number is preserved at the renormalizable level. If the string scale and the scale of massive exotics is beyond 10 TeV or so, this will also make the nonrenormalizable contributions to lepton number violation unobservable
- The $\mathcal{U}, \mathcal{D}$ quarks as well as the electron singlet associated with $\mathcal{L}$ remain massless.

Products of Gepner models typically have large discrete symmetries. These might be broken by the simple-current extensions procedure, as well as turning on closed string moduli. It is however expected that in subspaces of the moduli space there are remnants of the discrete symmetry. As the previous analysis shows, such symmetries are crucial for the phenomenological viability of this class of vacua and their presence must be carefully analyzed but this is beyond the scope of the present paper.

## 6. The D-terms

The general form of the $D$-term potential is

$$
\begin{equation*}
V_{D}=\sum_{i} D_{i}^{2} \tag{6.1}
\end{equation*}
$$

For the $\mathrm{U}(1)$ 's the D -term has the general form

$$
\begin{equation*}
D_{i}=\xi_{i}+\sum\left(q_{i}\left|X_{i}\right|^{2}\right) \tag{6.2}
\end{equation*}
$$

where $q_{i}$ is the charge of the chiral superfield $X_{i}$ under the corresponding gauge group $\mathrm{U}(1)_{i}$, and $\xi_{i}$ is the associated FI term. For the three U(1)'s of the model we have

$$
\begin{align*}
D_{a} & =\xi_{a}+Q^{I} Q^{I \dagger}-U^{i} U^{i \dagger}-D^{i} D^{i \dagger}-\mathcal{U}^{i} \mathcal{U}^{i \dagger}-\mathcal{D D}^{\dagger},  \tag{6.3}\\
D_{c} & =\xi_{c}-E^{I} E^{I \dagger}-U^{i} U^{i \dagger}+D^{i} D^{i \dagger}+H H^{\dagger}+\mathcal{L \mathcal { L } ^ { \dagger } - \overline { H H ^ { \dagger } } + N N ^ { \dagger } ,}  \tag{6.4}\\
D_{d} & =\xi_{d}-E^{I} E^{I \dagger}+L^{i} L^{i \dagger}-\mathcal{U}^{i} \mathcal{U}^{i \dagger}+\mathcal{D D ^ { \dagger }}-N N^{\dagger} \tag{6.5}
\end{align*}
$$

The contribution from non - abelian D terms to the Higgs potential has the standard from

$$
\begin{equation*}
D_{S U(2)}^{2}=\frac{g^{2}}{8}\left(H H^{\dagger}-{\overline{H H^{\prime}}}^{\dagger}\right)^{2}+\frac{g^{2}}{2}\left(H \bar{H}^{\dagger}\right)\left(\bar{H} H^{\dagger}\right) \tag{6.6}
\end{equation*}
$$

Finally the $D$-term potential is

$$
\begin{equation*}
V_{D}=D_{a}^{2}+D_{c}^{2}+D_{d}^{2}+D_{S U(2)}^{2} \tag{6.7}
\end{equation*}
$$

### 6.1 The Fayet-Iliopoulos terms

An important ingredient for the phenomenology of orbifold models is the presence and size of FI terms. FI terms can appear at disk level, and their presence is typically tracked by a spontaneous breaking of the associated $\mathrm{U}(1)$ global symmetry due to the D-term potential they generate. An important question is whether a FI term can appear at one loop if it is zero at tree level. This was answered in the negative in [33] where a calculation of the FI term was performed in the $Z_{3}$ orientifold, and was argued to hold for more general orbifolds. This was confirmed in the case of intersection $D_{6}$ branes in a flat background, 34. However it is not obvious that such a conclusion holds more generally for the RCFT vacua that we study here.

Consider a general orientifold ground state based on an arbitrary CFT and its BCFT. We assume that the CFT and BCFT realize a ground state with $\mathrm{N}=1$ spacetime fourdimensional supersymmetry. Moreover, all consistency conditions are satisfied at tree-level (sphere and disk) and the disk tadpoles have been canceled. All such assumptions are valid in the vacua we are considering made out of RCFTs including Gepner models.

Consider the $\mathrm{U}(1)$ gauge groups in this ground state that may be anomalous, but are massless at tree level (the mass developed by anomalous $\mathrm{U}(1)$ 's is a annulus effect [26].) This by definition implies that their associated FI term is zero at disk order as it would otherwise break the gauge symmetry or supersymmetry at tree level. We will now show that no FI term can be generated at one loop.

To track a non-zero FI term at one loop we may calculate the one-loop mass term of scalars charged under the $\mathrm{U}(1)$ in question. Such scalars were massless at tree level.

There are three diagrams at one loop that contribute to the mass term of such scalars. The first is an annulus diagram with the two scalar vertex operators inserted on the same boundary. The second is a Moebius diagram with the two scalar vertex operators inserted


Figure 1: Annulus and Moebius diagrams with the two scalars inserted on the same boundary and their UV factorization.


Figure 2: Annulus diagram with the two scalars inserted on opposite boundaries and its UV factorization.
on the only boundary of the surface. Both of these are drawn in figure 11. The third diagram is shown in figure 2 and involves an annulus with the two vertex operators inserted on opposite boundaries. ${ }^{10}$

All of these diagrams are very similar in structure to the ones we must consider in order to calculate the mass of anomalous $\mathrm{U}(1)$ gauge bosons in orientifolds [26]. In such diagrams, the two vertex operators give a kinematical piece that is $\mathcal{O}\left(p^{2}\right)$. Therefore, to obtain a contribution to the mass that is $\mathcal{O}(1)$ as the momentum is small $\left(p^{2} \rightarrow 0\right)$, we must obtain an $1 / p^{2}$ pole from the integration over the moduli of the surface. There are two corners such divergent terms can appear. In the open-string IR channel, this divergence is logarithmic at best (or finite). The only source of the pole is in the UV, and it is a contact term. It can be obtained by going to the transverse closed string channel and then looking at a massless divergence. At that limit the diagrams factorize as shown in figures 1 and 2 .

For the two diagrams of figure 1 the residue of the $1 / p^{2}$ pole is given by a product of a tree-level three-point coupling that couples a scalar and its conjugate to a massless closed string mode, and the sum of the disk level tadpoles. Therefore, if tadpoles cancel at tree level this contribution is identically zero.

On the other hand, in the diagram of figure 2 the residue of the $1 / p^{2}$ pole is a product of two disk two point functions, each of them mixing the charged opens-string scalar to a closed string massless state. However, if the $\mathrm{U}(1)$ symmetry is intact at tree level such two-point mixing terms are identically zero.

[^7]Therefore, there are no one-loop corrections to FI terms in orientifold vacua under the conditions spelled out earlier. It should be noted that as the arguments above assume the background of CFT, they are not automatically applicable to vacua that contain RR fluxes. ${ }^{11}$

Once the one loop correction is zero no further perturbative or non-perturbative corrections are expected.

Returning to our vacuum, we deduce that the FI are zero at the Gepner point, but they may be non-zero if we move in some directions of the closed string moduli space. The moduli along these directions are in the same chiral multiplets as the axions that cancel the anomalies of the relevant $\mathrm{U}(1)$ symmetries. Therefore it is necessary to not move in these directions.

## 7. Instanton corrections

As we have already seen, there is a remaining problem towards the phenomenological viability of the string vacuum under study, namely that there is no source for the masses of the $\mathcal{U}, \mathcal{D}$ quarks and the $\mathcal{L}$ leptons: a whole family is so far massless.

The missing couplings violate the charge conservation of the two anomalous $\mathrm{U}(1)$ symmetries. We expect that instanton effects (both gauge instantons and stringy instantons) must non-perturbatively violate these symmetries. This is therefore a source for the missing couplings.

Spacetime instantons in string theory have been analyzed for the first time after the advent of non-perturbative duality symmetries, (see 35 for a review). Their study has obtained a boost recently [36] as it became obvious that they are crucial for several phenomenological questions in orientifold vacua, from generating neutrino masses to Yukawa couplings to triggering supersymmetry breaking.

In our case to generate the relevant terms needed we need two kinds of instantons: one that violates $\left(\mathrm{U}(1)_{c}, \mathrm{U}(1)_{d}\right)$ charges by $(-1,1)$ units that we will call $I$ and a conjugate one $I^{*}$ that violates charges by $(1,-1)$ units. ${ }^{12}$ In the case they may generate the following non-perturbative superpotential up to cubic order (further details are beyond the scope of the paper)

$$
\begin{gather*}
W_{I}^{n p}=Q U L+Q \mathcal{D} \bar{H}+L L E+L \bar{H} \quad, \quad W_{I^{*} I^{*}}^{n p}=N N+N N R  \tag{7.1}\\
W_{I^{*}}^{n p}=Q \mathcal{U} K+E K K+K \bar{H} N+N+N R+N R R \tag{7.2}
\end{gather*}
$$

It is important to arrange that the instantons do not violate the $Z_{2}$ discrete symmetry, in which case the surviving non-perturbative superpotential reads

$$
\begin{equation*}
W^{n p}=Q \mathcal{D} \bar{H}+Q \mathcal{U} H+E \mathcal{L} H+\mathcal{L} N \bar{H}+N N+N R \tag{7.3}
\end{equation*}
$$

and as expected provides Yukawa couplings for $\mathcal{U}, \mathcal{D}$ quarks, the $\mathcal{L}$ lepton and the neutrinos.

[^8]|  | $L^{1}$ | $L^{2}$ | $\mathcal{L}$ | $N$ | $R^{1}$ | $R^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $L^{1}$ | 0 | 0 | 0 | $v$ | $v e^{-S}$ | $v e^{-S}$ |
| $L^{2}$ | 0 | 0 | 0 | $v$ | $v e^{-S}$ | $v e^{-S}$ |
| $\mathcal{L}$ | 0 | 0 | $\frac{v^{2}}{M_{s}}$ | $v e^{-S}$ | $v$ | $v$ |
| $N$ | $v$ | $v$ | $v e^{-S}$ | $M_{s} e^{-2 S}$ | $M_{s} e^{-S}$ | $M_{s} e^{-S}$ |
| $R^{1}$ | $v e^{-S}$ | $v e^{-S}$ | $v$ | $M_{s} e^{-S}$ | $M_{s}$ | $M_{s}$ |
| $R^{2}$ | $v e^{-S}$ | $v e^{-S}$ | $v$ | $M_{s} e^{-S}$ | $M_{s}$ | $M_{s}$ |

Table 7: Order of magnitude estimates of the Neutrino mass matrix elements. $v$ stands for the Higgs vev, $M_{s}$ is the string scale, and $e^{-S}$ stands for an instanton contribution.

In principle one can search for boundary states with the required number of zero-modes to produce the required stringy instantons. However, there are several complicating issues that have to be dealt with, such as the fact that we are not in the exact RCFT point (which may lead to differences in the number of non-chiral zero-modes), the postulated $Z_{2}$ symmetry, the possibility that undesired zero-modes may be lifted by fluxes, which we cannot take into account in the present formalism, the fact that tree-level couplings between physical fields and zero-modes are needed, plus the fact that not all boundary states present in the continuum may be accessible within the context of RCFT. For this reason a negative result would not be conclusive anyway, and we will not investigate this further in the present paper, but take as our working hypothesis that the required instanton corrections exist.

## 8. Neutrino masses

An important ingredient in any realisation of the Standard Model is whether neutrino masses near what is measured today are possible. A favourite mechanism for generating such neutrino masses is the see-saw mechanism and as we will see a version of this mechanism is possible in our vacuum.

We will recollect here the superpotential that is relevant for neutrino masses from (5.2), (5.3), (5.4) and (7.3). It includes both renormalizable and non-renormalizable contributions as well as non-perturbative effects.

$$
\begin{equation*}
W_{\nu}=R R+L N \bar{H}+\mathcal{L} \bar{H} R+\mathcal{L} \mathcal{L} \bar{H}^{2}+L \bar{H} R+\mathcal{L} \bar{H} N+N N+N R \tag{8.1}
\end{equation*}
$$

The order of magnitude of the contributions of each term in the superpotential to the neutrino mass matrix is summarized in table 7. In this table the Higgs vev is labeled as v, the string scale $M_{s}$ is expected to be near the unification scale, and the instanton factors are sketchily labeled $e^{-S}$ and they can be small.

It is a straightforward numerical exercise to verify that a matrix such as that in table 7 can reproduce neutrino masses as suggested by experiment with $\mathrm{O}(1)$ coefficients. ${ }^{13}$

[^9]
## 9. Gauge couplings and unification

In orientifold models the hypercharge is given by ${ }^{14}$

$$
\begin{equation*}
Y=\sum_{i} k_{i} Q_{i} \tag{9.1}
\end{equation*}
$$

where $Q_{i}$ are the overall $\mathrm{U}(1)$ generators of $\mathrm{U}\left(N_{i}\right)$ groups coming from complex brane stacks. From this we can determine, [1], 5], the hypercharge coupling constant in the standard field theory normalization as follows

$$
\begin{equation*}
\frac{1}{g_{Y}^{2}}=\sum_{i} \frac{2 N_{i}}{g_{i}^{2}} \tag{9.2}
\end{equation*}
$$

where $g_{i}$ is the gauge coupling of $i$-th stack, at the string scale. These are determined at the tree-level by the string coupling and other moduli, like volumes of longitudinal dimensions as well as potential internal magnetic fields. At higher orders, they also receive string threshold corrections.

For our vacuum with the hypercharge embedding (3.1) we obtain

$$
\begin{equation*}
\frac{1}{g_{Y}^{2}}=\frac{1}{6 g_{a}^{2}}+\frac{1}{2 g_{c}^{2}}+\frac{1}{2 g_{d}^{2}} \tag{9.3}
\end{equation*}
$$

from which we may compute the $\sin ^{2} \theta_{W}$ at the string scale

$$
\begin{equation*}
\sin ^{2} \theta_{W} \equiv \frac{g_{Y}^{2}}{g_{b}^{2}+g_{Y}^{2}}=\frac{1}{1+\frac{g_{b}^{2}}{6 g_{a}^{2}}+\frac{g_{b}^{2}}{2 g_{c}^{2}}+\frac{g_{b}^{2}}{2 g_{d}^{2}}} \tag{9.4}
\end{equation*}
$$

We have neglected stringy thresholds here, but they can be computed following [37].
At the Gepner point and at the string scale, $g_{a}=\frac{g_{b}}{\sqrt{2}}=g_{c}=g_{d}$. The extra factor for $g_{b}$ appears because the $b$ brane is a real brane and this changes the normalization of the gauge coupling. Also (9.4) gives

$$
\begin{equation*}
\sin ^{2} \theta_{W}\left(M_{s}\right)=\frac{3}{10} \tag{9.5}
\end{equation*}
$$

This value differs from the usual GUT value $3 / 8$ by $20 \%$,
As shown in appendix A, there is no scale at which the weak $(\mathrm{SU}(2))$ coupling constant can become twice the strong coupling constant as is the case at the Gepner point. This suggests that a correct fit to the SM gauge couplings is possible if in the appropriate position in moduli space, this relation is modified appropriately. The best case is that one moves to a point in moduli space where $g_{b}$ becomes equal to $g_{a}$. In such a case if we assume for example $g_{a}=g_{b}=g_{c}=g_{d}$ then

$$
\begin{equation*}
\sin ^{2} \theta_{W}\left(M_{s}\right)=\frac{6}{13} \tag{9.6}
\end{equation*}
$$

that differs from $3 / 8$ by about $20 \%$. In this case we show in appendix $A$ that the standard unification ratio can be adjusted by lowering the mass scale of non-chiral exotic multiplets below the string scale. Of course several other intermediate possibilities are also allowed.

[^10]
## 10. On supersymmetry breaking via gaugino condensation

The hidden sector gauge group, $\mathrm{SU}(2)$ has a coupling that becomes strong provided its chiral multiplets have masses close to the string scale. This will drive gaugino condensation and can break supersymmetry.

At the scale where the $\mathrm{SU}(2)_{h}$ gauge group becomes strongly coupled the corresponding gaugino condensate can trigger the supersymmetry breaking [38]-[40] (see [41] [42] for a review). In particular the supersymmetry breaking terms in the low energy effective action have the form $\frac{1}{M_{s t r}} \int d^{2} \theta W^{\alpha} W_{\alpha} \Phi \Phi$, where $W_{\alpha}$ is a chiral superfield whose lower component is the gaugino $\lambda_{\alpha}$ and $\Phi$ is a matter chiral superfield. ${ }^{15}$ After the gaugino condensate develops a vacuum expectation value the mass term of the form $\frac{\langle\lambda \lambda\rangle}{M_{s t r}^{2}}$ can be generated. The value of the gaugino condensate is related to the scale $\Lambda$ as $<\lambda \lambda>\sim \Lambda^{3}$ (an exact relation for the case of $\operatorname{SU}(2)$ gauge group can be found in [13]). From this relation, we must have $\Lambda \sim 10^{11.7} \mathrm{GeV}$ in order to have a supersymmetry breaking scale of the correct magnitude.

To estimate a scale where the hidden sector gauge group $\mathrm{SU}(2)$ becomes strongly coupled we use the equation

$$
\begin{equation*}
\Lambda=M_{s} e^{\frac{1}{2 b_{h} \alpha\left(M_{s}\right)}}, \tag{10.1}
\end{equation*}
$$

where

$$
\begin{equation*}
b_{h}=2 N_{T}+\frac{1}{2} N_{X}-6, \tag{10.2}
\end{equation*}
$$

and $N_{T}$ and $N_{X}$ are number of the chiral superfields $T$ and $X$ from the hidden sector which contribute to the corresponding one loop beta-function. We take $\tilde{\alpha}^{-1}\left(M_{\text {str. }}\right) \sim 323.5$. One can consider different values for $b_{h}$. Let us first take the case that no chiral superfields contribute to $b_{h},\left(N_{T}=N_{X}=0\right)$ i.e., one has only the contribution from the gauge bosons. One gets $\Lambda \sim 10^{4.2} \mathrm{GeV}$. Another case is when one $X$-field contributes to $b_{h}$ $\left(N_{T}=0, N_{X}=1\right)$. In this case one has $\Lambda \sim 10^{3.2} \mathrm{GeV}$. If there is a contribution from more than one field $X$, the corresponding value of $\Lambda$ will lie below the scale $M_{Z}$.

To obtain a high enough value of the gaugino condensation scale thresholds of KK states must be invoked. A direct computation shows that if the compactification scale is of the order of $10^{15} \mathrm{GeV}$, KK descendants of the $\mathrm{SU}(2)$ vector multiplet will drive the $\mathrm{SU}(2)$ coupling strong at $\Lambda \sim 10^{11.7} \mathrm{GeV}$.

## 11. Chiral symmetry breaking in the hidden sector

The vacuum discussed here has a spectrum tabulated in table 7. In particular, the hidden sector $\mathrm{SU}(2)_{h}$ has a chiral multiplet in the adjoint as well as 4 multiplets in the fundamental, half of them carrying $Y=\frac{1}{2}$ and the other half $Y=-\frac{1}{2}$. Neglecting for the moment the

[^11]SM interactions, there is an $\operatorname{SU}(4)$ chiral symmetry (the fundamental representation of $\mathrm{SU}(2)$ is pseudoreal).

If we label the $4 \mathrm{SU}(2)_{h}$ doublet fermions by $X_{a, \alpha}^{I}$ where $a$ is the $\mathrm{SU}(2)_{h}$ spinor index, $\alpha=1,2$ is the spin index and $I=1,2,3,4$ is a flavor index, then $Y\left(X_{\alpha}^{1,2}\right)=\frac{1}{2}, Y\left(X_{\alpha}^{3,4}\right)=$ $-\frac{1}{2}$. A gauge invariant order parameter for chiral symmetry breaking is

$$
\begin{equation*}
Z^{I J}=X_{a, \alpha}^{I} X_{b, \beta}^{J} \epsilon^{\alpha \beta} \epsilon^{a b} \quad, \quad Z^{I J}=-Z^{J I} \tag{11.1}
\end{equation*}
$$

and its expectation value breaks chiral symmetry $\mathrm{SU}(4) \rightarrow \mathrm{Sp}(4)$ 50.
The alignment of the chiral condensate is however crucial concerning the (spontaneous) breaking of $\mathrm{U}(1)_{c}$ and eventually electromagnetism. As the limits on the photon mass are very stringent, this issue is of crucial importance in assessing the viability of this string vacuum. The hypercharge of $Z^{12}$ is $Y=1$, that of $Z^{34}$ is $Y=-1$ while the other four $Z^{I J}$ have $Y=0$.

As in technicolor, the effective potential is generated by the exchange of the SM gauge bosons and it will prefer a direction where the $\mathrm{U}(1)_{c}$ is unbroken, 50]. As such directions exist, and are given by $Z^{12}=Z^{34}=0$, we conclude that for massless $X$ fields, $\mathrm{U}(1)_{e m}$ remains unbroken. If we now move in moduli space, so that the X multiplets obtain an $\mathrm{SU}(4)$ invariant mass, we are guaranteed to remain at the same minimum and $\mathrm{U}(1)_{e m}$ is still expected to remain unbroken.

So far our discussion above assumes the absence of supersymmetry. In the presence of unbroken supersymmetry, the potential for vacuum alignment due to the gauge interactions or masses is identically zero because of supersymmetry. However, if eventually supersymmetry is broken at a low scale then the potential discussed in the non-supersymmetric case resurfaces and our earlier conclusions are valid.

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## A. Analysis of the gauge couplings

In this appendix we give a brief analysis of the renormalization group equations for gauge coupling constants (see also [46] for a similar discussion). In particular we will show that if the couplings at string scale are related by a relation similar to that of the Gepner point point, $g_{a}=\frac{g_{b}}{\sqrt{2}}=g_{c}=g_{d}$, (which in particular implies $\sin ^{2} \theta_{w}=\frac{3}{10}$ ) then there is no way
of fitting to the low energy coupling constants of the standard model. In particular we will derive an upper bound for the weak coupling constant for this this to be possible. We will then investigate another relation at the string scale, namely $g_{a}=g_{b}=g_{c}=g_{d}$, (which in particular implies $\sin ^{2} \theta_{w}=\frac{6}{13}$ ) which as we show, fits the SM couplings, if some of the non-chiral exotics have masses below the string scale. In general as we vary the appropriate closed string moduli, the couplings at the string scale will generically vary, and the two relations we investigate here are two indicative cases.

We use the one-loop renormalization group equations

$$
\begin{equation*}
\frac{1}{\tilde{\alpha}_{i}(Q)}=\frac{1}{\tilde{\alpha}_{i}(\mu)}-2 b_{i} \log \frac{Q}{\mu}, \tag{A.1}
\end{equation*}
$$

where $\tilde{\alpha}_{1}=\frac{5}{3} \frac{g_{Y}^{2}}{16 \pi^{2}}, \tilde{\alpha}_{2}=\frac{g_{b}^{2}}{16 \pi^{2}}, \tilde{\alpha}_{3}=\frac{g_{3}^{2}}{16 \pi^{2}}$ and $g_{Y}^{2}, g_{b}^{2}$ and $g_{3}^{2}$ are the coupling constants of $\mathrm{U}(1)_{Y}, \mathrm{SU}(2)_{b}$ and $\mathrm{SU}(3)$ gauge groups. As we have mentioned before, we ignore the stringy threshold corrections in the renormalization group equation (A.1). The coefficients in the renormalization groups equations without taking into account the contribution of the hidden sector fields are 47]

$$
\begin{equation*}
b_{1}=\frac{4}{3} N_{\mathrm{Fam}}+\frac{1}{10} N_{\mathrm{Higgs}}, \quad b_{2}=-\frac{22}{3}+\frac{4}{3} N_{\mathrm{Fam}}+\frac{1}{6} N_{\mathrm{Higgs}}, \quad b_{3}=-11+\frac{4}{3} N_{\mathrm{Fam}}, \tag{A.2}
\end{equation*}
$$

for a case of a non-supersymmetric theory and

$$
\begin{equation*}
b_{1}=2 N_{\mathrm{Fam}}+\frac{3}{10} N_{\mathrm{Higgs}}, \quad b_{2}=-6+2 N_{\mathrm{Fam}}+\frac{1}{2} N_{\mathrm{Higgs}}, \quad b_{3}=-9+2 N_{\mathrm{Fam}} \tag{A.3}
\end{equation*}
$$

for the supersymmetric case theories. Here $N_{\text {Fam }}$ is a number of families of leptons and quarks and $N_{\text {Higgs }}$ is a number of Higgs (super)fields. Since we have three families and two Higgs (super)fields the values of the coefficients $b_{i}$ are

$$
\begin{equation*}
b_{1}=\frac{21}{5}, \quad b_{2}=-3, \quad b_{3}=-7 \tag{A.4}
\end{equation*}
$$

for energies below SUSY breaking scale and

$$
\begin{equation*}
b_{1}=\frac{33}{5}, \quad b_{2}=1, \quad b_{3}=-3 \tag{A.5}
\end{equation*}
$$

for energies above SUSY breaking scale (that we take to be equal to 1 TeV ). We assume that some of the non-chiral exotics acquire masses at an intermediate scale $M$ which is between the SUSY breaking scale and the string scale. Therefore these fields contribute to the running of the coupling constants at energies above the scale $M$. The corresponding contributions to the coefficients $b_{i}$ are

$$
\begin{equation*}
\Delta b_{1}=\frac{3}{10} N_{X}+\frac{2}{5} N_{Y_{s}}+\frac{3}{5} N_{P_{s}}, \quad \Delta b_{2}=0, \quad \Delta b_{3}=\frac{5}{2} N_{Y_{s}} \tag{A.6}
\end{equation*}
$$

where $N_{X}, N_{Y_{s}}$ and $N_{P_{s}}$ are the numbers of superfields $X, Y_{s}$ and $P_{s}$ which get masses at the scale $M$.

We can now estimate the value of the scale $M$ by fitting the gauge couplings to the observable values. Let us denote $3 g_{w}^{2} / 5 g_{Y}^{2}=\frac{3}{5} \operatorname{ctg}^{2} \theta_{W}\left(M_{s}\right) \equiv \gamma$. The renormalization group equation reads

$$
\begin{equation*}
\frac{1}{\tilde{\alpha}_{1}\left(M_{\text {susy }}\right)}+2 b_{1} \log \frac{M_{\text {susy }}}{M_{s}}+2 \Delta b_{1} \log \frac{M}{M_{s}}=\gamma\left(\frac{1}{\tilde{\alpha}_{2}\left(M_{\text {susy }}\right)}+2 b_{2} \log \frac{M_{\text {susy }}}{M_{s}}\right), \tag{A.7}
\end{equation*}
$$

From the equation (A.7) we observe that not all possible values of $\gamma$ are allowed, since the value of $\Delta b_{1} \log \frac{M_{s}}{M}$ must be positive. ${ }^{16}$ The limiting value of $\gamma$ corresponds to the case of the "standard" unification of coupling constants i.e., $\gamma=1$ and $\Delta b_{1}=0$. Therefore $\gamma$ must be less or equal to 1 . On the other hand $\Delta b_{1} \log \frac{M_{s}}{M}$ can not be too large, since it will imply that the value of the scale $M$ is very low. Estimating the lowest possible value of $M$ to be around 1 TeV we get the lowest value of $\gamma$ to be $\sim 0.26$ (this corresponds to the maximal value of $\Delta b_{1}$ ). Therefore we conclude that the value of $\gamma$ must be between 0.26 and 1.

Therefore we conclude that the case $g_{a}=\frac{g_{b}}{\sqrt{2}}=g_{c}=g_{d}$ is excluded since in this case $\gamma=\frac{7}{5}$. On the other hand for the case $g_{a}=g_{b}=g_{c}=g_{d}$ is allowed since $\gamma=\frac{21}{30}$. Let us consider this case in more detail. The renormalization group equation now reads

$$
\begin{equation*}
\frac{1}{\tilde{\alpha}_{1}\left(M_{\text {susy }}\right)}+2 b_{1} \log \frac{M_{\text {susy }}}{M_{s}}+2 \Delta b_{1} \log \frac{M}{M_{s}}=\frac{21}{30}\left(\frac{1}{\tilde{\alpha}_{2}\left(M_{\text {susy }}\right)}+2 b_{2} \log \frac{M_{\text {susy }}}{M_{s}}\right), \tag{A.8}
\end{equation*}
$$

where the coefficients $b_{i}$ and $\Delta b_{i}$ are given by (A.5) and (A.6) and the values of $\tilde{\alpha}_{1}\left(M_{\text {susy }}\right)$ and $\tilde{\alpha}_{2}\left(M_{\text {susy }}\right)$ can be obtained from (A.1), (A.4) and their values at $M_{Z}\left(\sim 10^{2} \mathrm{Gev}\right)$ scale (see for example [48]-49)

$$
\begin{equation*}
\frac{5}{3} \frac{g_{Y}^{2}\left(M_{Z}\right)}{4 \pi}=0.017, \quad \frac{g_{b}^{2}\left(M_{Z}\right)}{4 \pi}=0.034, \quad \frac{g_{3}^{2}\left(M_{Z}\right)}{4 \pi}=0.118 \tag{A.9}
\end{equation*}
$$

From the equation (A.8) one obtains (we have taken $M_{s} \sim 10^{16} \mathrm{GeV}$ )

$$
\begin{equation*}
\Delta b_{1} \log \frac{M_{s}}{M}=49.14 \tag{A.10}
\end{equation*}
$$

Obviously the value $\Delta b_{1}$ and therefore the value of the scale $M$ depends on how many and which superfields from the hidden sector contribute to the running of the coupling constant $g_{Y}^{2}$ between scales $M$ and $M_{s}$. For example let us consider the case when all non-chiral exotics contribute to the running of the coupling constant, i.e,. $N_{Y_{s}}=4, N_{X}=4, N_{P_{s}}=2$. This gives $\Delta b_{1}=4$, therefore $\log \frac{M_{s}}{M}=12.3$ and $M \sim 4.5 \times 10^{10} \mathrm{GeV}$. Let us note that this case will also change the running of the strong coupling constant comparing to the usual MSSM because of $Y_{s}$ field. Another possible case is when fields $Y_{s}$ obtain their masses at the string scale, i.e., $N_{Y_{s}}=0, N_{X}=4, N_{P_{s}}=2$. One has $\Delta b_{1}=2.4, \log \frac{M_{s}}{M}=20.5$ and $M \sim 1.25 \times 10^{7} \mathrm{GeV}$. Another example is $N_{Y_{s}}=1, N_{X}=1, N_{P_{s}}=2$. In this case one has $\Delta b_{1}=1.9$ and $M \sim 5.8 \times 10^{4} \mathrm{GeV}$.

Therefore one can conclude that if some of the hidden sector fields obtain their masses at an intermediate scale $M$ which is between SUSY breaking scale and the string scale, one can have a correct fitting of gauge coupling constants at the string scale, which is compatible with their low energy values.

[^12]
## B. Minimisation of the Higgs potential

As it was explained in the section 5, the bosonic component of the linear combination $H^{1}+H^{2}$ is expected to develop a vacuum expectation value and will be therefore identified with the Higgs field $H_{u}$. Because of the presence of singlets $R$ the Higgs potential is different from that of the MSSM and we analyze its minimization here.

Ignoring the terms which come from fourth order terms in the superpotential (like $H H \overline{H H})$ the relevant part of the potential has the form

$$
\begin{align*}
V= & m_{1}^{2} H H^{\dagger}+m_{2}^{2} \overline{H H^{\dagger}}+m_{3}^{2}\left(H \bar{H}-\bar{H}^{\dagger} H^{\dagger}\right)+\frac{g^{2}}{8}\left(H H^{\dagger}-\overline{H H}^{\dagger}\right)^{2}  \tag{B.1}\\
& +\frac{g^{2}}{2}\left(H \bar{H}^{\dagger}\right)\left(\bar{H} H^{\dagger}\right)+\eta^{2}(H \bar{H})\left(\bar{H}^{\dagger} H^{\dagger}\right)+\frac{g_{Y}^{2}}{8}\left(\xi_{Y}+H H^{\dagger}-\overline{H H}^{\dagger}\right)^{2},
\end{align*}
$$

where the term proportional to the parameter $\eta$ comes from the terms of the type $H \bar{H} R$ in the superpotential. Let us further take an ansatz for the Higgs fields as

$$
\begin{equation*}
H_{1}=v_{u}, \quad \bar{H}_{2}=v_{d} . \tag{B.2}
\end{equation*}
$$

The extremization conditions are

$$
\begin{align*}
& \left(m_{1}^{2}+\frac{\xi g_{Y}^{2}}{4}\right) v_{u}-m_{3}^{2} v_{d}+\frac{g^{2}+g_{Y}^{2}}{4}\left(v_{u}^{2}-v_{d}^{2}\right) v_{u}+\eta v_{u} v_{d}^{2}=0,  \tag{B.3}\\
& \left(m_{2}^{2}-\frac{\xi g_{Y}^{2}}{4}\right) v_{d}-m_{3}^{2} v_{u}-\frac{g^{2}+g_{Y}^{2}}{4}\left(v_{u}^{2}-v_{d}^{2}\right) v_{d}+\eta v_{u}^{2} v_{d}=0 . \tag{B.4}
\end{align*}
$$

Introducing the parametrization $v_{u}=v \cos \beta$ and $v_{d}=v \sin \beta$ we can solve the last two equations

$$
\begin{align*}
v^{2} & =-4 \frac{\tilde{m}_{1}^{2}-\tilde{m}_{2}^{2} \tan ^{2} \beta}{\left(g^{2}+g_{Y}^{2}\right)\left(1-\tan ^{2} \beta\right)+8 \eta^{2} \sin ^{2} \beta}  \tag{B.5}\\
\sin 2 \beta & =\frac{2 m_{3}^{2}}{\tilde{m}_{1}^{2}+\tilde{m}_{2}^{2}+\eta^{2}} \tag{B.6}
\end{align*}
$$

where we have denoted $\tilde{m}_{1}^{2}=m_{1}^{2}+\frac{\xi g_{Y}^{2}}{4}$ and $\tilde{m}_{2}^{2}=m_{2}^{2}-\frac{\xi g_{Y}^{2}}{4}$. The gauge symmetry breaking condition (i.e., the conditions that the solution (B.5)-(B.6) is the minimum) are $\tilde{m}_{1}^{2} \tilde{m}_{2}^{2}<m_{3}^{4}$ and $2 m_{3}^{2}<\tilde{m}_{1}^{2}+\tilde{m}_{2}^{2}+\eta^{2}$

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[^0]:    ${ }^{1}$ The search performed in was not complete, but it rather focused on finding the largest possible number of chirally distinct examples

[^1]:    ${ }^{2}$ This would involve acting with all 720 permutations on all MIPFs, but this is not completely straightforward. First one has to work out how the permutation acts on resolved fixed points, i.e. distinct fields that come from the same combinations of minimal model primaries.

[^2]:    ${ }^{3}$ Note that in 8 they were ordered according to frequency

[^3]:    ${ }^{4}$ There is another interesting possibility: that we choose as such singlets the fist string level descendants of the T multiplets. As the projection alternates between the string levels these will be global singlets. In this case in the spectrum No. 1 of table 2 we may consider an extra three right-handed neutrino singlets, two of the R type and one of the T type. In spectra Nos. 4 and 7 , all such neutrino singlets are of type T
    ${ }^{5}$ In cases where large internal volume is present, even a smaller number of right-handed neutrinos can be phenomenologically acceptable. This works via the presence and mixing of suitably light KK states and as shown in [2] it is not far from the current data of the neutrino sector. Finally, even in the complete absence of right-handed neutrino candidates, neutrino masses and mixings can be generated by higher dimension operators mediated by instantons 36.

[^4]:    ${ }^{6}$ As observed in [15, 8] this condition seems to be the strongest constraint towards finding a SM-like vacuum in Gepner orientifolds.

[^5]:    ${ }^{7}$ Our conventions are the $I, I, K=1,2,3, i, j, k=1,2$.
    ${ }^{8}$ There is also the possibility that string instantons break it, but we will not further entertain this possibility here.

[^6]:    ${ }^{9}$ There is the further possibility that $L^{i}$ also participate in electro-weak symmetry breaking. In that case the $\mathcal{U}$ quark has a tree-level Yukawa coupling.

[^7]:    ${ }^{10}$ This diagram was not considered in the early analysis of 33.

[^8]:    ${ }^{11}$ They seem though to be valid perturbatively in the RR field insertions.
    ${ }^{12}$ This cannot be the anti-instanton of I, as supersymmetry forbids the generation of superpotential couplings in that case.

[^9]:    ${ }^{13}$ We thank P. Anastasopoulos for doing this calculation.

[^10]:    ${ }^{14}$ We neglect here the possibility that traceless generators appear in the hypercharge. This happens many times, [8], but is not relevant for the vacua studied here.

[^11]:    ${ }^{15}$ Am alternative mechanism of the supersymmetry breaking via the gaugino condensate has been suggested in 44-45 in the framework of the brane world scenario. In these models the Standard Model gauge fields are propagating in the bulk, while the matter is localized on the brane. The value of the mass terms in these models depend on the size of the extra dimension.

[^12]:    ${ }^{16}$ It is in principal possible that stringy thresholds can bypass this constraint.

